

Non SUSY Unification in Left-Right Models.

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Abstract

We explore in a model independent way the possibility of achieving the non supersymmetric gauge coupling unification within left-right symmetric models, with the minimal particle content at the left-right mass scale which could be as low as 1 TeV in a variety of models, and with a unification scale M in the range $10^5 \text{ GeV} < M < 10^{17.7} \text{ GeV}$.

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1 Introduction

It has been known for more than a decade[1] that if we let the three gauge couplings $c_i\alpha_i^{-1}$ run through the “desert” from low to high energies, they do not merge together into a single point, where $\{c_1, c_2, c_3\} = \{\frac{3}{5}, 1, 1\}$ are the normalization constants of the Standard Model (SM) factors $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, respectively, embedded into $SU(5)$ [2]. This odd result claims for new physics at intermediate energy scales as for example:

1. The inclusion of the minimal supersymmetric (SUSY) partners of the SM fields at an energy scale $M_{susy} \sim 1 \text{ TeV}$, related to an unification scale $M \sim 10^{16} \text{ GeV}$ [3].
2. The inclusion of a minimal Left Right Symmetric Model (LRSM) at a mass scale $M_R \sim 10^{11} \text{ GeV}$, related to an unification scale $M \sim 10^{15} \text{ GeV}$ [4] in an $SO(10)$ Grand Unified Theory (GUT) [5].
3. The inclusion of the SUSY partners of the minimal LRSM at an energy scale $M_{susy} \sim M_R \sim 1 \text{ TeV}$, related to a unification scale $M \sim 10^{16} \text{ GeV}$ [6]. Etc..

The alternative approach, namely, to normalize the gauge couplings $c_i\alpha_i^{-1}$ to non-orthodox c_i , ($i = 1, 2, 3$) values was presented by these authors, in Ref. [7] for non-SUSY models, and in Ref. [8] for the SUSY ones, for possible GUT models which can descend in one single step to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \equiv G_{SM}$.

In this paper we present a systematic analysis of all the possible GUT models which descend in two steps to G_{SM} , with the LRSM as the intermediate step, paying special attention to those models with low M_R scale. The paper is organized in the following way: In section II we present the renormalization group equation formalism for the LRSM; in section III we carry our model independent analysis, and in section IV we present our results and conclusions. A technical appendix at the end gives the c_i , $i = 1, 2, 3$ values for most of the GUT models in the literature.

2 The Renormalization Group Equations

In a field theory, the couplings are defined as effective values, which are energy scale dependent according to the renormalization group equations. In the modified minimal substraction scheme [9], which we adopt in what follows, the one-loop renormalization group equations are

$$\mu \frac{d\alpha_i}{d\mu} \simeq -b_i \alpha_i^2, \quad (1)$$

where μ is the energy at which the coupling constants $\alpha_i = g_i^2/4\pi$, ($i = 1, 2, 3$) are evaluated, with g_1 , g_2 , and g_3 the gauge couplings of the SM factors $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively. The constants b_i are completely determined by the particle content in the model by

$$4\pi b_i = \frac{11}{3}C_i(vectors) - \frac{2}{3}C_i(fermions) - \frac{1}{3}C_i(scalars),$$

being $C_i(\dots)$ the index of the representation to which the (\dots) particles are assigned, and where we are considering Weyl fermion and complex scalar fields [10]. The boundary conditions for these equations are determined by the relationships

$$\alpha_{em}^{-1} = \alpha_1^{-1} + \alpha_2^{-1}, \quad \text{and} \quad \tan^2 \theta_W = \frac{\alpha_1}{\alpha_2}, \quad (2)$$

which at the electroweak scale imply

$$\alpha_1^{-1}(m_Z) = \frac{1 - \sin^2 \theta_W(m_Z)}{\alpha_{em}(m_Z)}, \quad \text{and} \quad \alpha_2^{-1}(m_Z) = \frac{\sin^2 \theta_W(m_Z)}{\alpha_{em}(m_Z)}. \quad (3)$$

Combining those expressions with the experimental values

$$\alpha_{em}^{-1}(m_Z) = 127.90 \pm 0.09 \text{ [11, 12]},$$

$$\begin{aligned}\sin^2 \theta_W(m_Z) &= 0.2312 \pm 0.00017 \text{ [11, 12]} \quad \text{and} \\ \alpha_3(m_Z) &= \alpha_s = 0.1191 \pm 0.0018 \text{ [11]},\end{aligned}\tag{4}$$

we get:

$$\begin{aligned}\alpha_1^{-1}(m_Z) &= 98.330 \pm 0.091, \\ \alpha_2^{-1}(m_Z) &= 29.571 \pm 0.043, \quad \text{and} \\ \alpha_3^{-1}(m_Z) &= 8.396 \pm 0.127.\end{aligned}\tag{5}$$

The unification of the three SM gauge couplings is properly achieved if they meet together into a common value $\alpha = g^2/4\pi$ at a certain energy scale M , where g is the gauge coupling constant of the unifying group G . However, since $G \supset G_{SM}$, the normalization of the generators corresponding to the subgroups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ is in general different for each particular group G , and therefore the SM coupling constants α_i differ at the unification scale from α by numerical factors c_i ($\alpha_i = c_i \alpha$). In $SU(5)$ these factors are [2] $\{c_1, c_2, c_3\} = \{\frac{3}{5}, 1, 1\}$ (we call them the canonical values), which are the same for $SO(10)$ [5], E_6 [13], $[SU(3)]^3 \times Z_3$ [14], $SO(18)$ [15], E_8 [16], $SU(15)$ [17], $SU(16)$ [18], and $SU(8) \otimes SU(8)$ [19], but they are different for other groups such as $SU(5) \otimes SU(5)$ [20], $[SU(6)]^3 \times Z_3$ [21], the Pati-Salam models [22], etc. (see Table I in the appendix).

The constants c_i can also be seen as a consequence of the affine levels (or Kac-Moody levels) at which the gauge factor G_i is realized in the effective four dimensional string [23], even if there is not an unification gauge group at all; but if it does, they are related to the fermion content of the irreducible representations of G . As a matter of fact, if α_i is the coupling constant of G_i , a simple group embedded into G , then

$$c_i \equiv \frac{\alpha_i}{\alpha} = \frac{Tr T^2}{Tr T_i^2},\tag{6}$$

where T is a generator of the subgroup G_i properly normalized over a representation R of G , and T_i is the same generator but normalized over the representation of G_i embedded into R (the traces run over complete representations). In this way for example, if just one standard doublet of $SU(2)_L$ is contained in the fundamental representation of G (plus any number of $SU(2)_L$ singlets), then $c_2 = 1$ (as in $SU(5)$); but this is not the general case. In this way we proof that for $i = 2, 3$, $c_i^{-1} = 1, 2, 3, \dots, n$ an integer number. The constants c_i are thus pure rational numbers satisfying $c_1 > 0$ and $0 < c_{2(3)} \leq 1$. They are fixed once we fix the unifying gauge structure. According to the table I in the appendix and in order to simplify matters, we are going to use for c_2 only the values 1 and $\frac{1}{3}$, and for c_3 the values 1 and $\frac{1}{2}$.

From Eqs. (2) and (6) it follows that at the unification scale the value of $\sin^2 \theta_W$ is given by

$$\sin^2 \theta_W \equiv \frac{\alpha_{em}}{\alpha_2} = \frac{c_1}{c_1 + c_2}. \quad (7)$$

Obviously, Eq. (7) is equivalent to that given in terms of the traces of the generators of $SU(2)_L$ and the electric charge for simple groups (see Ref. [2]). In order to connect this value at the scale M with the corresponding value at the scale m_Z the renormalization group equations (1) must be solved.

Our approach is now the following: we assume there are only three relevant mass scales m_Z , M_R , and M such that $m_Z < M_R < M$, where $m_Z \sim 10^2 \text{ GeV}$ is the electroweak mass scale, M_R is the mass scale where the LRSM (with and without discrete left-right (LR) symmetry) manifests itself, and M is the GUT scale. Then, the equations (1) must be solved, first for the energy range $m_Z < \mu < M_R$, and then for the range $M_R < \mu < M$, properly using at each stage the decoupling theorem [24].

Now for the energy interval $m_Z < \mu < M_R$, the one loop solutions to the equations (1) are:

$$\alpha_i^{-1}(m_Z) = \alpha_i^{-1}(M_R) - b_i(H) \ln \left(\frac{M_R}{m_Z} \right), \quad (8)$$

where the beta functions b_i ($i = 1, 2, 3$) are [10]

$$2\pi \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} - \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F - \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} H, \quad (9)$$

with $F = 3$ the number of families and H the number of low energy Higgs field doublets. Notice by the way that we are not including in the former equation the normalization factor $\frac{3}{5}$ into b_1 coming from $SU(5)$, and wrongly included in some general discussions. $H = 1$ in the SM; nevertheless, a general model can have more than one low energy Higgs field, and in principle H may be taken as a free parameter ($H = 2$ in the minimal supersymmetric model).

For the interval $M_R < \mu < M$, the evolution of the gauge couplings is dictated by the beta functions of the LRSM whose gauge group is [25] $G_{LR} \equiv SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, with the matter fields transforming as $\Psi_L = (3, 2, 1, 1/6) \oplus (\bar{3}, 1, 2, -1/6) \oplus (1, 2, 1, -1/2) \oplus (1, 1, 2, 1/2)$ for each generation, where the numbers between brackets label $(SU(3)_c, SU(2)_L, SU(2)_R, U(1)_{B-L})$ representations.

The LRSM is broken down spontaneously by the Higgs sector, which in general contains N_B bidoublet Higgs fields $\varphi(1, 2, 2, 0)$, N_{TL} triplets in the representation $\Delta_L(1, 3, 1, 1)$, N_{TR} triplets in

the representation $\Delta_R(1, 1, 3, -1)$, N_{DL} doublets in the representation $\phi_L(1, 2, 1, -1/2)$, and N_{DR} doublets in the representation $\phi_R(1, 1, 2, 1/2)$. In the so called minimal LRSM[25], $N_{TR} = N_B = 1$ and $N_{DL} = N_{DR} = N_{TL} = 0$, but in general N_{TL} , N_{TR} , N_{DL} , N_{DR} and N_B should be taken as free parameters to be fixed by the specific model.

In a general context, the vacuum expectation values that may be used to break the symmetry are $\langle \Delta_R^0 \rangle \sim \langle \phi_R^0 \rangle \sim M_R$ (Δ_R^0 represent the electromagnetic neutral direction in Δ_R , etc.), $\langle \varphi^0 \rangle \sim \langle \phi_L^0 \rangle \sim m_Z$, and $\langle \Delta_L^0 \rangle = 0$. It then follows that $H = 2N_B + N_{DL}$.

The discrete LR symmetry implies invariance under the exchange $L \leftrightarrow R$ in the model (this is the so called D parity) with the consequence that $g_{2L} = g_{2R}$ for the energy interval $M_R < \mu < M$. This symmetry is respected by the gauge and the fermion content of any LRSM, but it is broken by the scalar sector as it is shown anon.

Indeed, the Higgs field scalars can drastically alter the solution to the renormalization group equations, and in order to make any definite statement about the mass scales in a particular model, we must know which components of the Higgs representations have masses of order m_Z , M_R and M . However, to know the masses of the scalars is equivalent to the hopeless task of knowing the values of all the coupling constants appearing in the scalar potential (with radiative corrections included). So, in order to guess what the real effect of the scalars is, the so called extended survival hypothesis was introduced in Ref. [26]. Basically the hypothesis consists in assuming that only the components of the Higgs representations which are required for the breaking of a particular symmetry are the only ones which are not superheavy. In other words: “scalar Higgs fields acquire the maximum mass compatible with the pattern of symmetry breaking” (for a more detailed explanation and application to SO(10), see Ref. [26]).

The one loop solutions to Eqs. (1) for the energy interval $M_R < \mu < M$ are:

$$\alpha_i^{-1}(M_R) = \frac{1}{c_i} \alpha^{-1} - b'_i(N_B, N_{TL}, N_{TR}, N_{DL}, N_{DR}) \ln \left(\frac{M}{M_R} \right), \quad (10)$$

where $i = BL, 2L, 2R, 3$. The beta functions b'_i are now: $b'_3 = b_3 = 7/2\pi$ (with the assumption that no low energy colored scalars exist (as demanded by the extended survival hypothesis); if they do, they may cause a too fast proton decay, and spoil the asymptotic freedom for $SU(3)_c$); and b'_{2R} , b'_{2L} and b'_{BL} given by:

$$2\pi \begin{pmatrix} b'_{BL} \\ b'_{2L} \\ b'_{2R} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ \frac{22}{3} \end{pmatrix} - \begin{pmatrix} \frac{8}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F - \begin{pmatrix} 0 \\ \frac{N_B}{3} \\ \frac{N_B}{3} \end{pmatrix} - \begin{pmatrix} N_{TL} + N_{TR} \\ \frac{2N_{TL}}{3} \\ \frac{2N_{TR}}{3} \end{pmatrix} - \begin{pmatrix} \frac{N_{DL} + N_{DR}}{6} \\ \frac{N_{DL}}{6} \\ \frac{N_{DR}}{6} \end{pmatrix}. \quad (11)$$

From Eqs. (10) and (11) we get $g_{2L} = g_{2R}$ if $N_{TL} = N_{TR}$ and $N_{DL} = N_{DR}$. But if $N_{TR} \neq N_{TL} = 0$ as demanded by the extended survival hypothesis, then one could only have exact left-right symmetry at the GUT scale.

The hypercharge Y of the SM is given by

$$Y = T_{3R} + Y_{B-L}, \quad (12)$$

which implies the relation $\alpha_1^{-1}(M_R) = \alpha_{2R}^{-1}(M_R) + \alpha_{BL}^{-1}(M_R)$. Then the beta function for $U(1)_Y$ for the energy interval $M_R < \mu < M$ may be written as $b'_1 = b'_{2R} + b'_{BL}$ with $c_1^{-1} = c_{2R}^{-1} + c_{BL}^{-1}$, and $c_{2R} = c_{2L} = c_2$, ($c_{BL}^{-1} = \frac{2}{3}$ for the minimal fermion field content of the LRSM). These relations together with Eqs. (8) and (10) allow us to write:

$$\begin{aligned} \alpha_1^{-1}(m_Z) &= \frac{1}{c_1} \alpha^{-1} + \left(\frac{40+H}{12\pi} \right) \ln \left(\frac{M}{m_Z} \right) - \frac{1}{6\pi} (22 - 3N_{TL} - 5N_{TR} - N_{DR}) \ln \left(\frac{M}{M_R} \right) \\ \alpha_2^{-1}(m_Z) &= \frac{1}{c_2} \alpha^{-1} - \frac{1}{12\pi} \left[(20 - H) \ln \left(\frac{M}{m_Z} \right) - 4N_{TL} \ln \left(\frac{M}{M_R} \right) \right] \\ \alpha_3^{-1}(m_Z) &= \frac{1}{c_3} \alpha^{-1} - \frac{7}{2\pi} \ln \left(\frac{M}{m_Z} \right), \end{aligned} \quad (13)$$

which is a system of 3 equations with 3 unknowns: α , M_R and M ($m_Z = 91.187 \pm 0.007$ GeV[11] and $\alpha_i^{-1}(m_Z)$ as in Eqs. (5) are taken as inputs). c_i ($i = 1, 2, 3$), $N_B, N_{TL}, N_{TR}, N_{DL}$, and N_{DR} ($H = 2N_B + N_{DL}$) are model dependent parameters. Evidently, there is always solution to the system of equations in (13), but the consistency of the unification scheme demands that $m_Z < M_R < M \leq 10^{19}$ GeV $\sim M_P$ (the Planck Mass). When we solve Eqs. (13) for the minimal LRSM ($N_{TR} = N_B = 1$, $N_{TL} = N_{DL} = N_{DR} = 0$) for the canonical values ($\{c_1, c_2, c_3\} = \{\frac{3}{5}, 1, 1\}$) we get $M = 2.5 \cdot 10^{16}$ GeV, $M_R = 2.7 \cdot 10^9$ GeV and $\alpha^{-1} = 45.45$.

Notice that if $N_{TL} = 0$ (as demanded by the extended survival hypothesis), the last two equations in (13) are independent of M_R , and they are enough to fix the GUT scale M (and α of course). If we solve them for $c_2 = 1$, $c_3 = \frac{1}{2}$ (one family models with chiral color[27], as for example $SU(5) \otimes SU(5)$ [20], $SO(10) \otimes SO(10)$ [28]), we get for $H < 22$ the unphysical solution $M \gg M_P$. A further analysis shows that for $22 < H < 30$ we get $M_P > M > 10^{16}$ GeV which in turn implies $M_R < m_Z$ which is also unphysical. To get $M_R > 1$ TeV requires for those models $H > 40$ which gives $M < 10^{12}$ GeV in serious conflict with proton decay which is always present in those models. So the two step breaking pattern $SU(5) \otimes SU(5) \longrightarrow G_{LR} \longrightarrow G_{SM}$ is not allowed (the one step $SU(5) \otimes SU(5) \longrightarrow G_{SM}$ is also forbidden[7, 8]). This conclusion is valid even for the case $g_{2L} \neq g_{2R}$ at the GUT scale, a variant of the model introduced in the second paper of Ref. [20]. Similar conclusions follow for $SO(10) \otimes SO(10)$ [28]. To use $N_{TL} \neq 0$ makes things even worse.

When we solve Eqs.(13) for $c_2 = \frac{1}{3}$, $c_3 = 1$ (models with three families and vector-like color as for example $[SU(6)]^3 \times Z_3$ [21]) we get $M \simeq 5m_Z$, an unacceptable solution. So the two step breaking pattern $[SU(6)]^3 \times Z_3 \longrightarrow G_{LR} \longrightarrow G_{SM}$ is not allowed either (the one step breaking pattern $[SU(6)]^3 \times Z_3 \longrightarrow G_{SM}$ is also forbidden for this group[7, 8]).

So our analysis makes sense only for two cases: $\{c_2, c_3\} = \{1, 1\}$ (one family models with vector like-color), and $\{c_2, c_3\} = \{\frac{1}{3}, \frac{1}{2}\}$ (models with 3 families and chiral color). In what follows we are going to refer only to these situations.

Before moving to a general analysis, let us see for example what happens for $SO(10) \longrightarrow G_{LR} \longrightarrow G_{SM}$. As mentioned above, $\{c_1, c_2, c_3\} = \{\frac{3}{5}, 1, 1\}$, and there are not exotic fermions in the spinorial 16 representation used for the matter fields, but the scalar content is not quite uniquely defined, and there are as many versions of the model as you wish. A couple of examples are:

1- In Ref.[4] the following symmetry breaking pattern is implemented:

$$SO(10) \xrightarrow{\phi_{(210)}} G_{LR} \xrightarrow{\phi_{(126)}} G_{SM} \xrightarrow{2\phi_{(10)}} SU(3)_c \otimes U(1)_{EM}.$$

ϕ_{210} gets mass at the GUT scale and it does not contribute to the renormalization group equations. For ϕ_{126} , $\Delta_R = \Delta_L = 1$, but only $\langle \Delta_R \rangle \neq 0$. For the final breaking only one $\phi_{(10)}$ is needed, but at least two must be used in order to achieve proper isospin breaking. Then $N_{TR} = 1$, $N_B = 2$, $N_{TL} = N_{DL} = N_{DR} = 0$. We get $M = 2.0 \cdot 10^{15}$ GeV, $M_R = 1.6 \cdot 10^{11}$ GeV and $\alpha^{-1} = 42.6$.

2- A more recent version of (SUSY) $S0(10)$ implements the breaking with the following scalar content[29]:

$$SO(10) \xrightarrow{\phi_{(45)}} G_{LR} \xrightarrow{\phi_{(16+cc)}} G_{SM} \xrightarrow{2\phi_{(10+16+cc)}} SU(3)_c \otimes U(1)_{EM}.$$

With the extended survival hypothesis in mind we have $N_{TL} = N_{TR} = 0$, $N_B = N_{DL} = N_{DR} = 2$. We get $M = 2.2 \cdot 10^{14}$ GeV, $M_R = 9 \cdot 10^{12}$ GeV, and $\alpha^{-1} = 40.16$. In both examples the D parity is broken below the GUT scale.

Since the scalar sector is the most obscure part of any gauge theory, it is clear that, N_i ($i = B, TL, TR, DL$ and DR) can be taken as free parameters, resulting in all sort of models for all sort of tastes. Since the Higgs field scalars can drastically change the GUT scales, we can not state with confidence neat values for M and M_R . We elaborate on this in the next section.

Before proceeding to our model independent analysis let us mention that we are going to consider the possibility of adding arbitrary large numbers of scalars Higgs fields in order to get unification. In many cases this may result in the coupling constants becoming so large as to make the theory non-perturbative before unification is achieved. Even though the extended survival

hypothesis[26] greatly diminishes the effect of the Higgs scalar fields, we will pay special attention to our parameter space region in the analysis, in order not to run into non-perturbative regimes of the coupling constants. As a matter of fact, the assumption that no low energy colored scalars exist is all what is needed for the cases considered ahead.

3 Model Independent Analysis

In this section we are going to study two different situations. First we are going to reduce the freedom we have in our parameter space by imposing the extended survival hypothesis. Second, we reduce the freedom by restoring the D parity to the LRSM.

3.1 Solutions to the equations with extended survival hypothesis

If we impose the extended survival hypothesis as a constraint in the solutions to the renormalization group equations for the LRSM, we must set $N_{TL} = 0$. Then Eqs. (13) get reduced to a system of 3 equations with 3 unknowns, and the following set of parameters: c_i ($i = 1; 2; 3$); H , and $N'_T = 5N_{TR} + N_{DR}$. The solution of Eqs. (13) for M , M_R and α as functions of these parameters is:

$$\alpha^{-1} = \frac{42t_{32} - (20 - H)t_{23}}{D} \quad (14)$$

$$\ln\left(\frac{M}{m_z}\right) = \frac{12\pi}{D} [c_2\alpha_2^{-1}(m_Z) - c_3\alpha_3^{-1}(m_Z)] \quad (15)$$

and

$$\ln\left(\frac{M}{M_R}\right) = \frac{6\pi N}{c_1(22 - N'_T)D} \quad (16)$$

where $N = [(20 - H)(t_{21} - t_{23}) + (40 + H)(t_{12} - t_{13}) + 42(t_{32} - t_{31})]$, $D = 42c_3 - (20 - H)c_2$ and $t_{ij} = t_{ij}(m_Z) = c_i c_j \alpha_j^{-1}(m_Z)$. From Eq. (16) it can be seen that either $H < 7$ and $N'_T < 22$ ($N_{TR} < 5$), or $H > 7$ and $N'_T > 22$, in order to have $M_R \leq M$.

From Eqs. (15) and (16) we plot in Figure 1 the allowed region for H and N'_T that give unification, for the canonical values of c_i ; and in Figure 2 we plot c_1 Versus N'_T for $H = 2$ and $\{c_2; c_3\} = \{\frac{1}{3}; \frac{1}{2}\}$.

To analyze the implications of each one of the figures we must have in mind the following constraints:

1. $M \leq M_P \sim 10^{19}$ GeV, the Planck scale (actually $M \leq M_{max} \sim 10^{17.7}$ GeV, obtained when there is not contribution from the scalar sector).

2. $M > 10^5$ GeV in order to suppress unwanted flavor changing neutral currents [11, 30].
3. $M > 10^{16}$ if the proton is allowed to decay in the particular GUT model.
4. $8m_Z \leq M_R \leq M$. The lower limit is taken from the particle data book[11], the upper limit is imposed by consistency of the renormalization group equations.

3.1.1 Analysis of Figure 1.

The allowed region lies inside the lines $M_R = 8m_Z$ and $M_R = M$, but if the proton does decay in the model under consideration then the allowed region lies in the lower left corner between the lines $M = 10^{16}$ GeV, $M_R = 8m_Z$, $H = 0$ and $N'_T = 0$.

For GUT models with unstable proton (which are most of the models for the groups in the canonical entry in Table 1 in the appendix), $M_R \sim 1$ TeV is obtained for $H = 2$ and $N'_T = 13$ ($N_{TR} = 2$ and $N_{DR} = 3$), which in turn implies $M \sim 2.59 \cdot 10^{16}$ GeV.

For models in the canonical entry with a stable proton (as for example $[SU(3)]^3 \times Z_3$ [14] and $SU(8) \otimes SU(8)$ [19]) the allowed region is wider and divided in two regions. One for $H < 7$; $N'_T < 22$ and the other for $H > 7$ and $N_T > 22$. There are plenty of examples of models with $M_R \sim 1$ TeV for those situations.

3.1.2 Analysis of Figure 2.

The entire plane in figure 2 is related to the GUT scale $M \sim 10^8$ GeV (fixed just by the values of H , c_2 and c_3). The allowed region lies between the lines $M_R = M$ and $M_R = 8m_Z$. From the figure we see that a value of $c_1 = \frac{3}{19}$ crosses the $M_R = 8m_Z$ line at $N'_T = 31$ ($N_{TR} = 6$, $N_{DR} = 1$), which means that the model $[SU(6)]^4 \times Z_4$ [31] can have the following chain of spontaneous descent

$$[SU(6)]^4 \times Z_4 \xrightarrow{M} G_{LR} \xrightarrow{M_R} G_{SM} \xrightarrow{m_Z} SU(3)_c \otimes U(1)_{EM},$$

with $M \sim 10^8$ GeV and $M_R \sim 9m_Z$, as long as an irreducible representation of the GUT group with 6 right handed triplets is used to break G_{LR} down to the SM gauge group and then a representation of the GUT group with only two $SU(2)_L$ Higgs field doublets is used in the last breaking step.

A further look into the equations for this group shows that for $N'_T = 0$ and $H = 2, 3$ we get $M_R = M \sim 10^8$ GeV, meaning that a single step spontaneous descent is possible for this model with a very economical set of Higgs field scalars. But this result has been already published in Ref.[7]. Here we just confirm the published result.

3.2 Solutions to the equations with D parity

In order to restore the D parity in the renormalization group equations for the energy interval $M_R < \mu < M$ we must have $N_{TL} = N_{TR} \equiv N_T$ and $N_{DL} = N_{DR} = N_D$. Again we solve Eqs. (13) as a function of c_i , H , N_T and N_D . Using the equations we get, we plot in Figure 3 the allowed region for H and N_T that gives unification for the canonical values of c_i , and in Figure 4 we plot c_1 Versus N_T for $H = 2$, $N_D = 0$ and $\{c_2; c_3\} = \{1; 1\}$.

3.2.1 Analysis of Figure 3

For models with unstable proton the allowed tiny region lies in the lower left corner, between the lines $N_T = 0$, $H = 0$ and $M = 10^{16}$ GeV. From the figure we get $M_R > 10^9$ GeV, $N_T \leq 1$ and $H \leq 2$.

For models with an stable proton the allowed region is larger, with boundaries given by the lines $M_R = M$ and $M = 10^5$ GeV which excludes the possibility $M_R \sim$ a few TeV, unless $N_T > 50$ which is very unlikely in realistic models.

3.2.2 Analysis of Figure 4

The allowed region of parameters lies inside the lines $N_T = 0$, $M = M_R$ and $M = 10^{16}$ GeV for models with unstable proton, and inside the lines $N_T = 0$, $M = M_R$ and $M_R = 8 m_Z$ for models with an stable proton. As can be seen, the canonical value $c_1 = \frac{3}{5}$ lies inside both regions, but far from $M_R \sim 1$ TeV.

In general, large values for N_T are required ($H < 8$) in LRSM with D parity, in order not to have unduly large values for M_R .

4 Conclusions

To conclude let us emphasize that it is possible to unify the SM group using the LRSM as an intermediate stage for a variety of models, with $1 \text{ TeV} \leq M_R \leq 10^{15}$ GeV. From our study, three family models with vector like color are excluded (as $[SU(6)]^3 \times Z_3$), and one family models with chiral color are also excluded (as $SU(5) \otimes SU(5)$ [20], and $SO(10) \otimes SO(10)$ [28]).

We point out that in our analysis we have neglected threshold effects which depend on the particular structure of each model, and also we do not include second order corrections to the renormalization group equations which are typically of the order of the threshold effects. In others aspects it is completely general. Within this limitations we may conclude that it is indeed

possible to achieve the unification of the coupling constants of the SM in a general class of non supersymmetric models which have the minimal LRSM as an intermediate step, with an M_R scale as low as 1 TeV. We are aware that this class of models may suffer of hierarchy problems.

From our analysis we may extract the following morals:

- 1- Higgs scalars play a crucial role in the solution to the renormalization group equations.
- 2- It is simple to construct realistic non SUSY - GUT models with an intermediate Left-Right symmetry at a mass scale $M_R \sim 1$ TeV (just read them from the figures).
- 3- LRSM with D parity are quite different to those without D parity.
- 4- For low M_R , models with D parity are less realistic than models without D parity, in the sense that they make use of a very large amount of Higgs scalars.
- 5- It is impossible to sustain the D parity when the extended survival hypothesis is imposed.

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Appendix.

In this appendix we give the c_i $i = 1; 2; 3$ values for most of the GUT groups in the literature. They are presented in table I. The “Canonical” entry refers to the following groups: $SU(5)$ [2] $SO(10)$ [5], E_6 [13], $[SU(3)]^3 \times Z_3$ [14], $SU(15)$ [17], $SU(16)$ [18], $SU(8) \times SU(8)$ [19], E_8 [16], and $SO(18)$ [15]. Also, in the Canonical entry we have normalized the c_i values to the $SU(5)$ numbers; for example, the actual values for $SU(16)$ are: $\{c_1^{-1}; c_2^{-1}; c_3^{-1}\} = \{20/3; 4; 4\} = 4\{5/3; 1; 1\}$. This normalization makes sense because physical quantities such as $\sin^2\theta_W$, M_R and M depend only on ratios of two c_i values (see Eqs. (7), (15), and (16)).

Most of the groups in the first entry have the canonical values for c_i due to the fact that they contain $SU(5)$ via regular embeddings (see the table 58 in Ref.[32]), which do not change the rank of the corresponding group. For others as for example $SU(16)$ it is just an accident.

c_3^{-1} can take only the values 1, 2, 3, 4 for one family groups, or higher integer values for family groups. $c_3^{-1} = 1$ when it is $SU(3)_c$ which is embedded in the GUT group G ; $c_3^{-1} = 2$ when it is the chiral color [27] $SU(3)_{cL} \times SU(3)_{cR}$ which is embedded in G , etc. For example $c_3^{-1} = 4$ in $SU(16)$ due to the fact that the color group in the GUT group is $SU(3)_{cuR} \times SU(3)_{cdR} \times SU(3)_{cuL} \times SU(3)_{cdL}$.

Group	c_1^{-1}	c_2^{-1}	c_3^{-1}
Canonical	5/3	1	1
$SU(5) \otimes SU(5)$	13/3	1	2
$SO(10) \otimes SO(10)$	13/3	1	2
$[SU(6)]^3 \times Z_3$	14/3	3	1
$[SU(6)]^4 \times Z_4$	19/3	3	2
E_7	2/3	2	1
$[SU(4)]^3 \times Z_3$	11/3	1	1
$[SU(2F)]^4 \times Z_4$	$(9F - 8)/3$	F	2

Table 1: c_1, c_2 and c_3 values for most of the GUT models in the literature. The entry “canonical” is explained in the main text, and $F = 1, 2, \dots$ stand for the number of families in that particular model.

For family groups c_2^{-1} take the values $1, 2, \dots F$ for $1, 2, \dots F$ families. Indeed, the c_i values for the F family Pati-Salam models [33] $[SU(2F)]^4 \times Z_4$ are $\{c_1^{-1}; c_2^{-1}; c_3^{-1}\} = \{(9F - 8)/3; F; 2\}$.

In general, $c_{2(3)}^{-1} = 1, 2, \dots f$, where f is the number of fundamental representations of $SU(2)_L$ ($SU(3)_c$) contained in the fundamental representation of the GUT group. For example, $c_2^{-1} = 4$ in $SU(16)$ because the 16 representation of $SU(16)$ contains four $SU(2)_L$ doublets; three for $(u, d)_L$ and one for $(\nu_e, e)_L$.

The group $[SU(4)]^3 \times Z_3$ in Table I is not the vector-like color version of the two family Pati-Salam group, but it is the one family theory introduced in Ref. [28]. Also, the group $[SU(6)]^4 \times Z_4$ in the Table is not the 3 family Pati-Salam model, but a version of such model (with 3 families) without mirror fermions, introduced in Ref. [31].

All models in Table I are realistic, except E_7 [34] which is a two family model with the right handed quarks in $SU(2)_L$ doublets.

The values c_i (and Table I) are interesting by themselves because they are related to the Kac-Moody levels (κ_i) of String GUTs [23]. Indeed: $c_i^{-1} = \kappa_i$, $i = 1, 2, 3$. Curiously enough, the values for c_1 are integer multiple of $1/3$ for all the known groups, we do not know why.

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Figure captions:

Figure 1. Allowed values for H and N'_T for the canonical values $(c_1, c_2, c_3) = (\frac{3}{5}, 1, 1)$. Notice that the unification scale M is independent of the value for N'_T .

Figure 2. Allowed region for the parameters c_1 and N'_T for models with $c_2 = \frac{1}{3}$, $c_3 = \frac{1}{2}$ and $H = 2$. The cross represent the case $c_1 = \frac{3}{19}$ and $N'_T = 31$ discussed in the main text.

Figure 3. Allowed values for H and N_T for models with D parity at the M_R scale, and the canonical values $(c_1, c_2, c_3) = (\frac{3}{5}, 1, 1)$.

Figure 4. Allowed region for the parameters c_1 and N_T for models with $c_2 = c_3 = 1$, $H = 2$ and D parity above the M_R scale.

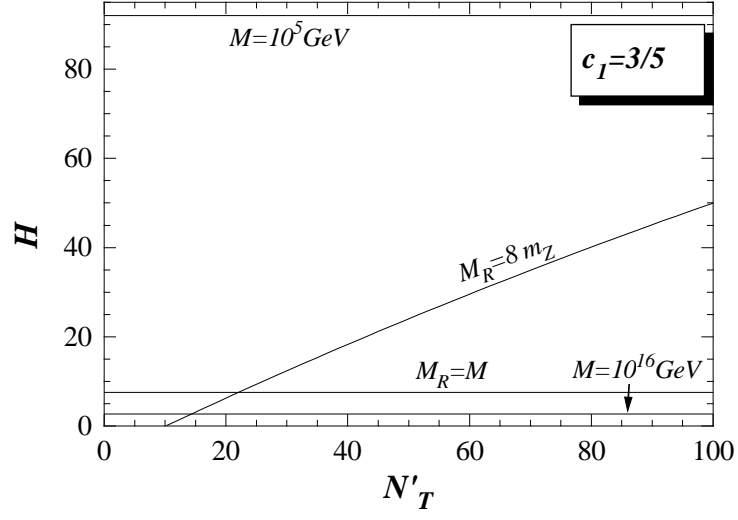


Figure 1:

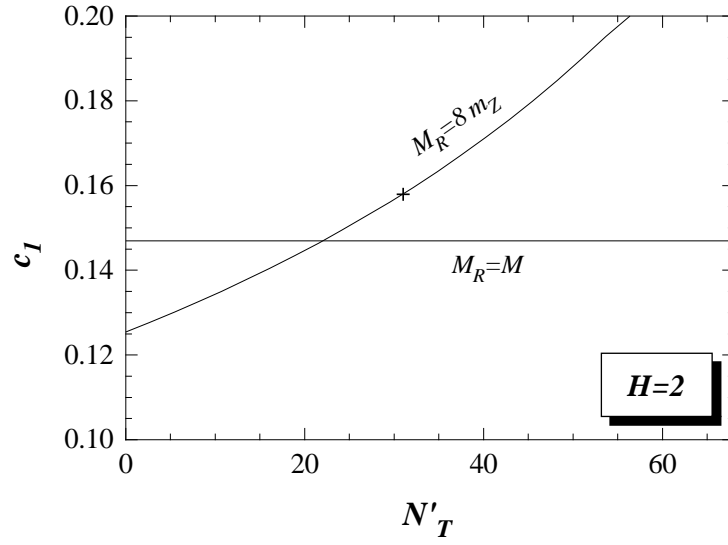


Figure 2:

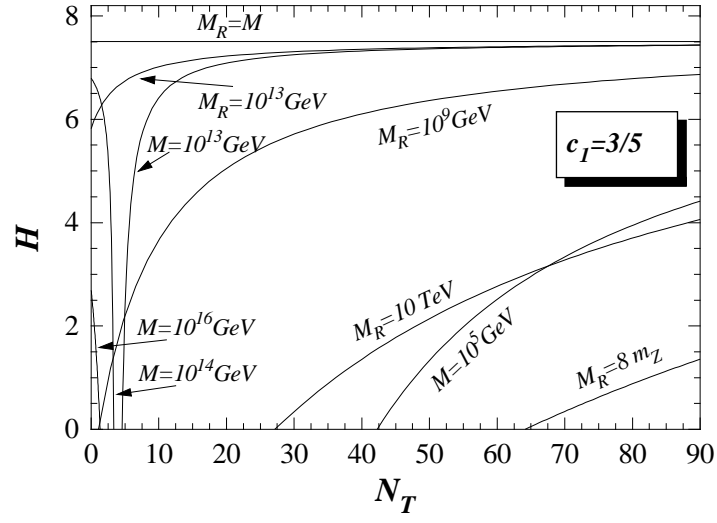


Figure 3:

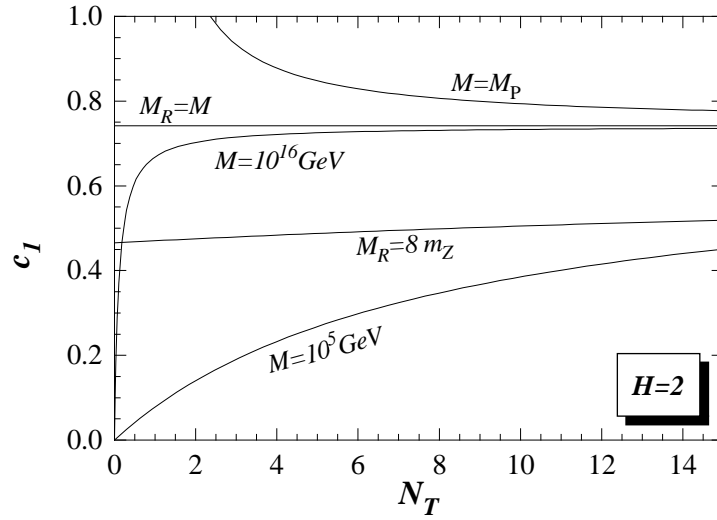


Figure 4: